Example 3g: Laminate Theory – Single Ply

This example problem employs the classical lamination theory capabilities of MAC/GMC 4.0 to simulate the response of a single ply SiC/Ti-21S laminate with several different fiber orientations. It is important to keep in mind that there are several major differences between analyzing a laminate and a composite material, even if both are composed of the same phases. First, while a composite material is a three-dimensional continuum point without edges and boundary influences, a laminate has a finite thickness and thus top and bottom boundaries. Further, classical lamination theory is based on the assumption that the laminate is in a state of plane stress at every point. That is, (in MAC/GMC's laminate coordinate system, Figure 1.2) the out-of-plane normal stress (σ_{zz} or σ_{33}) as well as the out-of-plane shear stresses σ_{xz} and σ_{yz} or σ_{13} and σ_{23}) are required to be zero throughout the laminate. Thus global loading that involves any out-of-plane stress (or strain) components is inadmissible. Finally, since the layers within the laminate are composed of composite materials, MAC/GMC 4.0 treats the laminate problem with an embedded multi-scale approach. The local behavior of the composite material in each layer is modeled using GMC, which is embedded within the lamination theory analysis that handles the structural (laminate) scale. For more information on the lamination theory and the code's laminate analysis capabilities, see the MAC/GMC 4.0 Theory Manual Section 3.

In the present example, a single 0.25 fiber volume fraction SiC/Ti-21S composite ply is considered in which the fiber orientation is varied between 0° (along the *x* loading direction) and 90° (transverse to the *x* loading direction). A simple 2×2 square fiber, square pack RUC architecture is employed, and a midplane strain of 0.02 is applied at a temperature of 650 °C.

MAC/GMC Input File: `example_3g.mac`

```
MAC/GMC 4.0 Example 3g - Lamination theory, single ply, varying fiber angle
*CONSTITUENTS
  NMATS=2
  M=1 CMOD=6 MATID=E
  M=2 CMOD=4 MATID=A
*LAMINATE
  NLY=1
  LY=1 MOD=2 THK=1. ANG=0 ARCHID=1 VF=0.25 F=1 M=2
#  LY=1 MOD=2 THK=1. ANG=15 ARCHID=1 VF=0.25 F=1 M=2
#  LY=1 MOD=2 THK=1. ANG=45 ARCHID=1 VF=0.25 F=1 M=2
#  LY=1 MOD=2 THK=1. ANG=30 ARCHID=1 VF=0.25 F=1 M=2
#  LY=1 MOD=2 THK=1. ANG=60 ARCHID=1 VF=0.25 F=1 M=2
#  LY=1 MOD=2 THK=1. ANG=75 ARCHID=1 VF=0.25 F=1 M=2
#  LY=1 MOD=2 THK=1. ANG=90 ARCHID=1 VF=0.25 F=1 M=2
*MECH
  LOP=1
  NPT=2 TI=0.,200. MAG=0.,0.02 MODE=1
*THERM
  NPT=2 TI=0.,200. TEMP=650.,650.
*SOLVER
  METHOD=1 NPT=2 TI=0.,200. STP=1.
*PRINT
  NPL=6
*XYPLOT
```

```
  FREQ=5
  LAMINATE=1
   NAME=example_3g X=1 Y=10
  MACRO=0
  MICRO=0
*END
```

Annotated Input Data

1) Flags: None

2) Constituent materials (***CONSTITUENTS**) [KM_2]:

Number of materials:	2	(NMATS=2)
Materials:	SiC fiber	(MATID=E)
	Ti-21S	(MATID=A)
Constitutive models:	SiC fiber: linearly elastic	(CMOD=6)
	Ti-21S matrix: Isotropic GVIPS	(CMOD=4)

3) Analysis type (***LAMINATE**) → Laminate Analysis [KM_3]:

Number of layers:	1	(NLY=1)
Layer analysis model:	Doubly periodic GMC	(MOD=2)
Layer thickness:	1.	(THK=1.)
Layer fiber angle:	0°	(ANG=0)
	15°	(ANG=15)
	30°	(ANG=30)
	45°	(ANG=45)
	60°	(ANG=60)
	75°	(ANG=75)
	90°	(ANG=90)
Architecture:	2×2, square fiber, square pack	(ARCHID=1)
Material assignment:	SiC fiber	(F=1)
	Ti-21S matrix	(M=2)

The information for each layer is specified on a separate line of the input file. In the present example, the laminate contains only one layer and thus requires only one line. The information on the line is similar to that which is specified under ***RUC** for repeating unit cell analysis, with the addition of the layer thickness (THK) and the layer fiber orientation angle (ANG). Note that the thickness values are arbitrary, but the laminate force resultants are integrated through the thickness and thus scale with the overall laminate thickness. In order to execute the code for the seven different fiber orientations, the appropriate lines in the input file must be commented and uncommented. For more information on the laminate analysis input requirements, see the MAC/GMC 4.0 Keywords Manual Section 3.

4) Loading:

a) Mechanical (***MECH**) [KM_4]:

Loading option:	1 (loading in the laminate x-direction)	(LOP=1)
Number of points:	2	(NPT=2)
Time points:	0., 200. sec.	(TI=0.,200.)
Load magnitude:	0., 0.02	(MAG=0.,0.02)
Loading mode:	midplane strain/curvature control	(MODE=1)

Rather than applying loads in the form of stress and strain components as is done in repeating unit cell analysis, in laminate analysis, stress/moment resultants and midplane strains/curvatures are applied. In addition, as mentioned above, only in-plane loading is admissible so the loading options are numbered differently. Loading options 1 – 3 are associated with applied in-plane force resultants or midplane strains. Loading options 4 – 6 are associated with applied in-plane moment resultants or midplane curvatures. For more information on mechanical loading in laminate analysis, see the MAC/GMC 4.0 Keywords Manual Section 4.

b) Thermal (**`*THERM`**) [KM_4]:

Number of points:	2	(`NPT=2`)
Time points:	0., 200. sec.	(`TI=0.,200.`)
Temperature points:	650., 650. °C	(`TEMP=650.,650.`)

c) Time integration (**`*SOLVER`**) [KM_4]:

Time integration method:	Forward Euler	(`METHOD=1`)
Number of points:	2	(`NPT=2`)
Time points:	0., 200. sec.	(`TI=0.,200.`)
Time step sizes:	1. sec.	(`STP=1.`)

5) Damage and Failure: None

6) Output:

a) Output file print level (**`*PRINT`**) [KM_6]:

Print level:	6	(`NPL=6`)

b) x-y plots (**`*XYPLOT`**) [KM_6]:

Frequency:	5	(`FREQ=5`)
Number of laminate plots:	1	(`LAMINATE=1`)
Laminate plot name:	example_3g	(`NAME=example_3g`)
Laminate plot x-y quantities:	ε_{xx}^{0}, N_{xx}	(`X=1 Y=10`)
Number of macro plots:	0	(`MACRO=0`)
Number of micro plots:	0	(`MICRO=0`)

Because this example problem involves a laminate, an additional level of x-y plot output is available above the macro and micro plot output available in repeating unit cell analysis. The number of these laminate scale plots is specified by `LAMINATE=`. "`_lam.data`" is appended to the laminate level plot names, thus, in this example, the x-y plot output is written to a file named "`example_3g_lam.data`". Note that, since different quantities are available for plotting on the scale of the laminate, the numbering scheme for `X=` and `Y=` is different than that used for the macro and micro plots. For more information on generating laminate scale x-y plot files, see the MAC/GMC Keywords Manual Section 6.

7) End of file keyword: (**`*END`**)

Results

The following is taken from the MAC/GMC 4.0 output file for this example problem for the 0° ply:

```
-------------------- EFFECTIVE PROPERTIES AT TEMPERATURE =   650.00 ---------------------------

                     * FOR LAYER NUMBER  1 *

 CG - Effective/Macro Stiffness Matrix

    0.3082E+05    0.1277E+05    0.1277E+05
    0.1277E+05    0.2517E+05    0.1305E+05
    0.1277E+05    0.1305E+05    0.2517E+05
                                                0.5360E+04
                                                              0.5719E+04
                                                                            0.5719E+04

 CI - Effective/Macro Compliance Matrix

    0.4488E-04   -0.1500E-04   -0.1500E-04
   -0.1500E-04    0.5935E-04   -0.2317E-04
   -0.1500E-04   -0.2317E-04    0.5935E-04
                                                0.1866E-03
                                                              0.1749E-03
                                                                            0.1749E-03

 Effective Engineering Moduli

     E11S= 0.2228E+05
     N12S= 0.3342
     E22S= 0.1685E+05
     N23S= 0.3903
     E33S= 0.1685E+05
     G23S= 0.5360E+04
     G13S= 0.5719E+04
     G12S= 0.5719E+04

 Effective Thermal Expansion Coefficients

    0.7452E-05    0.1105E-04    0.1105E-04


  Local Q Stiffness For Layer  1

      2.434E+04    6.150E+03    0.000E+00
      6.150E+03    1.840E+04    0.000E+00
      0.000E+00    0.000E+00    5.719E+03

  Global Q Stiffness For Layer  1

      2.434E+04    6.150E+03    0.000E+00
      6.150E+03    1.840E+04    0.000E+00
      0.000E+00    0.000E+00    5.719E+03
-----------------------------------------------------------------------------------------------


---------------------- LAMINATE RESULTS AT TEMPERATURE =   650.00 ------------------------------


  Laminate Axial Stiffness Matrix [A]

      2.434E+04    6.150E+03    0.000E+00
      6.150E+03    1.840E+04    0.000E+00
      0.000E+00    0.000E+00    5.719E+03

  Laminate Coupling Stiffness Matrix [B]

      0.000E+00    0.000E+00    0.000E+00
      0.000E+00    0.000E+00    0.000E+00
      0.000E+00    0.000E+00    0.000E+00
```

```
 Laminate Bending Stiffness Matrix [D]

     2.028E+03    5.125E+02    0.000E+00
     5.125E+02    1.534E+03    0.000E+00
     0.000E+00    0.000E+00    4.766E+02

 Laminate Engineering Constants (only valid for symmetric laminates)

    Exx=  2.228E+04
    Nxy=  3.342E-01
    Eyy=  1.685E+04
    Gxy=  5.719E+03



==============================================================================================
*****                          Section III:  Time-Based Output                           *****
==============================================================================================


    1 TIME:  1.0000D+00     TEMP:  6.5000D+02     TSTEP:  1.0000D+00
 ---------------------------------------------------------------------------------------------------
     FORCE(N), MOMENT(M):  2.2280D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
       STRAIN, CURVATURE:  1.0000D-04  -3.3418D-05   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
          INELASTIC N, M:  0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
            THERMAL N, M:  0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
     OUT-OF-PLANE STRAIN: -3.3418D-05   0.0000D+00   0.0000D+00

     ABD MATRIX:
       |  2.43355D+04   6.14987D+03   0.00000D+00   0.00000D+00   0.00000D+00   0.00000D+00  |
       |  6.14987D+03   1.84029D+04   0.00000D+00   0.00000D+00   0.00000D+00   0.00000D+00  |
       |  0.00000D+00   0.00000D+00   5.71872D+03   0.00000D+00   0.00000D+00   0.00000D+00  |
       |  0.00000D+00   0.00000D+00   0.00000D+00   2.02796D+03   5.12489D+02   0.00000D+00  |
       |  0.00000D+00   0.00000D+00   0.00000D+00   5.12489D+02   1.53357D+03   0.00000D+00  |
       |  0.00000D+00   0.00000D+00   0.00000D+00   0.00000D+00   0.00000D+00   4.76560D+02  |
```

As shown above, for laminate analysis, the output to the MAC/GMC 4.0 output file is different than for repeating unit cell analysis. For each layer, the local (in principal material coordinates) and global (in laminate coordinates) reduced stiffness matrices (**Q**) are output. Further, the axial, coupling, and bending stiffness matrices (**A**, **B**, and **D**) for the laminate are output, as are the effective (or apparent) engineering properties for the laminate. The time-based output has changed as well. Instead of printing the six components of stress and strain at each time step (as is done for repeating unit cell analysis), for laminate analysis, the three components of force and moment resultant and the three components of midplane strain and curvature are printed. In place of the inelastic and thermal strains, inelastic and thermal force and moment resultants are output. Also, due to the plane stress assumption inherent within classical lamination theory, the out-of-plane stress components are zero throughout the laminate. However, the out-of-plane strain components may or may not (due to Poisson effects) be zero. Thus, these out-of-plane strain components are printed at each time step. Finally, rather than the composite stiffness, the laminate **ABD** matrix is output at each time step. Note that the above description pertains to a print level of 6 (`NPL=6`). More or less data is written to the MAC/GMC 4.0 output file if this value is altered (see the Keywords Manual Section 6 for details on the code's print levels).

Figure 3.15 shows that the [0°] ply, with its fibers oriented along the loading direction, gives the stiffest response. In fact, because a ply thickness of 1. was employed and no bending occurs, the response of the [0°] ply shown in Figure 3.15 is identical to the longitudinal stress-strain response generated for the SiC/Ti-21S composite at 650 °C in Example 1d (Figure 1.5). As the orientation angle of the ply rises from 0°, the response rapidly becomes more compliant as one might expect. However, at the higher angles this trend reverses – the most compliant response is exhibited by the [60°] ply. Then, as the orientation angle continues to rise to 75° and 90°, the ply's response stiffens. This trend is somewhat counterintuitive as one might expect the response of a [90°] ply, with its fibers oriented completely normal to the loading direction, to be the most compliant orientation. However, as stated by Jones (1974), "the extremum material properties [of a ply] do not necessarily occur in the principal material directions." Indeed, consider the formula for the apparent stiffness of an off-axis ply (Jones, 1975),

$$\frac{1}{E_{xx}} = \frac{\cos^4(\theta)}{E_{11}} + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_{11}}\right)\sin^2(\theta)\cos^2(\theta) + \frac{\sin^4(\theta)}{E_{22}}$$

where E_{xx} is the apparent stiffness in the x-direction, E_{11} and E_{22} are the longitudinal and transverse ply stiffness (i.e., principal material coordinate stiffnesses), G_{12} is the ply axial shear modulus, ν_{12} is the ply axial Poisson ration, and θ is the fiber orientation angle. For the SiC/Ti-21S composite considered in the present example, the ply properties in principal material coordinates (E_{11}, E_{22}, G_{12}, and ν_{12}) can be obtained from the output file:

```
Effective Engineering Moduli

     E11S=  0.223E+05
     N12S=  0.334E+00
     E22S=  0.168E+05
     N23S=  0.390E+00
     E33S=  0.168E+05
     G23S=  0.536E+04
     G13S=  0.572E+04
     G12S=  0.572E+04
```

Utilizing these values in conjunction with the formula for the apparent ply stiffness (given above) yields a plot of the apparent ply stiffness vs. fiber orientation angle for the 0.25 fiber volume fraction SiC/Ti-21S ply as shown in Figure 3.16. Clearly, the apparent stiffness of the ply first decreases, and then increases as the fiber orientation angle rises. The apparent modulus reaches a minimum at an angle of 55.6 °. Thus the trend shown in Figure 3.15, while somewhat counterintuitive, is correct.

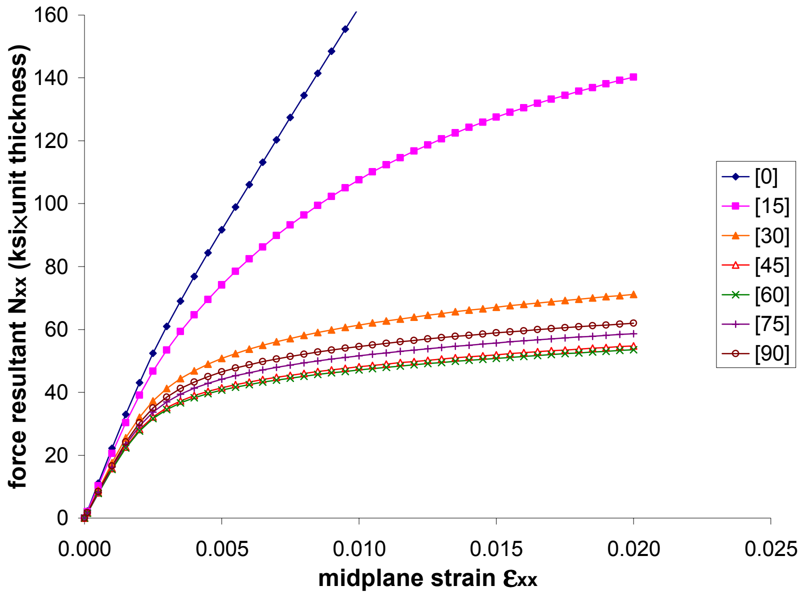

Figure 3.15 Example 3g: plot of the global tensile force resultant – midplane strain ($N_{xx} - \varepsilon_{xx}^{0}$) response for a 0.25 fiber volume fraction single ply SiC/Ti-21S laminate at 650 °C with varying fiber orientation angle.

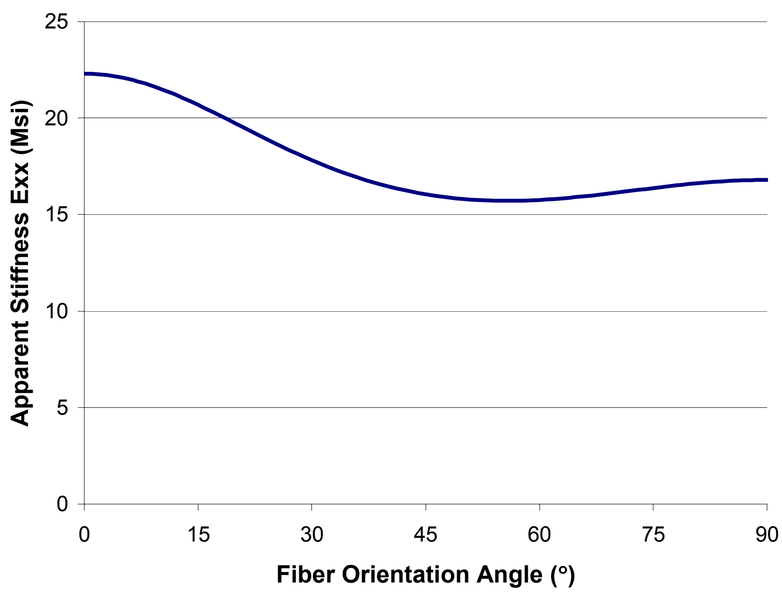

Figure 3.16 Example 3g: plot of the apparent stiffness of a 0.25 fiber volume fraction SiC/Ti-21S ply at 650 °C as a function of fiber orientation angle.